

Is there supercurvature mode of massive vector field in open inflation?

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Abstract. We investigate the Euclidean vacuum mode functions of a massive vector field in a spatially open chart of de Sitter spacetime. In the one-bubble open inflationary scenario that naturally predicts a negative spatial curvature after a quantum tunneling, it is known that a light scalar field has the so-called supercurvature mode, i.e. an additional discrete mode which describes fluctuations over scales larger than the spatial curvature scale. If such supercurvature modes exist for a vector field with a sufficiently light mass, then they would decay slower and easily survive the inflationary era. However, the existence of supercurvature mode strongly depends on details of the system. To clarify whether a massive vector field has supercurvature modes, we consider a U(1) gauge field with gauge and conformal invariances spontaneously broken through the Higgs mechanism, and present explicit expressions for the Euclidean vacuum mode functions. We find that, for any values of the vector field mass, there is no supercurvature mode. In the massless limit, the absence of supercurvature modes in the scalar sector stems from the gauge symmetry.

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1 Introduction

Recent observational data provide a strong support of the existence of extragalactic magnetic fields, in the range of $\mathcal{O}(10^{-14}\text{--}10^{-20})$ G on Mpc scales [1–10]. The generation of the magnetic field in high-redshift galaxies, clusters, and even in empty intergalactic region is still an unresolved problem in cosmology. No promising astrophysical process to generate the sufficient amount of the magnetic field on the large scales are known. As for the inflationary magnetogenesis, though the various mechanism are proposed, the several difficulties such as the strong coupling problem, the backreaction problem and the curvature perturbation problem in some specific models prevent successful production of magnetic field [11–13]. Actually both upper and lower limits on the inflation energy scale can be derived from these problems in model independent ways and the limits are considerably severe if the extragalactic magnetic fields are stronger than 10^{-16} G at present [14–16]. Thus it is known to be very difficult to generate the magnetic field in the context of the inflationary magnetogenesis on the flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe.

The superadiabatic growth of the magnetic fields in the open FLRW universe has been discussed in the literatures [17, 18]. The authors of these literatures assumed the existence of supercurvature modes of the magnetic field, which describes the fluctuations with the wavelength exceeding the spatial curvature scale. If a supercurvature mode exists, it decays slower than $1/a^2$, where a corresponds to the conventional scale factor of a FLRW universe, and can easily survive the inflationary era. Hence the relatively large amount of the magnetic

field on supercurvature scales would remain at late time. However, the existence of supercurvature modes of magnetic fields is non-trivial and should be critically studied. Adamek *et al.* [34] recently pointed out that the equations of motion of a U(1) gauge field with unbroken conformal and gauge symmetries can be rewritten in the form that is identical to those of massive scalar fields for which there is no supercurvature mode¹.

The purpose of the present paper is to investigate whether supercurvature modes exist for a massive vector field, in both scalar and vector sectors of the physical spectrum. To be specific, we consider a U(1) gauge field with both gauge and conformal symmetries spontaneously broken through the Higgs mechanism. As for the background geometry, we consider a de Sitter spacetime in the open chart. This is relevant to the one-bubble open inflation scenario that naturally predicts the spatially negative curvature universe. While the recent observational data show that the universe is almost exactly flat with accuracy of about 1%, $|1 - \Omega_0| \leq 10^{-2}$ [21], open inflation scenario is attracting a renewed interest in the context of the string landscape scenario [22, 23]. There are a huge number of metastable de Sitter vacua and the tunneling transition generally occurs through the nucleation of a true vacuum bubble in the false vacuum background. Because of the symmetry of the instanton solution, a bubble formed by the Coleman-De Luccia (CDL) instanton [24, 25] looks like an infinite open universe from the viewpoint of an observer inside. If the universe experienced a sufficiently long inflation after the bubble nucleation, then the universe becomes almost exactly flat and subsequently evolves as a slightly open FLRW universe. This leads to a natural realization of one-bubble open inflation (see e.g. [26–30]) and can be tested against observations [31–33].

This paper is organized as follows. We first illustrate the background spacetime in section 2. In section 3, we expand the U(1) gauge field by harmonic functions and write down the reduced action for the even and odd modes of the U(1) gauge field. In order to investigate the existence/absence of supercurvature modes, we show the quantization conditions for the even and odd modes on a Cauchy surface. With the obtained normalization conditions, we then analyze whether the supercurvature modes, which are normalizable on the Cauchy surface, exist in section 4. In section 5, as a consistency check, we explicitly calculate the Wightman function in the decoupling limit by using the (subcurvature) mode functions derived in section 3. It is shown that the correct expression for the Euclidean Wightman function is recovered in the decoupling limit without need for any supercurvature modes. Finally, section 6 is devoted to a summary and discussions.

2 Background

In this paper, we consider a U(1) gauge field with both gauge and conformal symmetries spontaneously broken through the Higgs mechanism in an open de Sitter geometry, i.e. a de Sitter spacetime in the open chart.

Before showing relevant forms of the background metric, we illustrate that this setup is appropriate to investigate the existence/absence of supercurvature mode of a massive vector field in open inflationary universe. Let us begin with a system which consists of multi scalar fields and the U(1) gauge field minimally coupled with Einstein gravity. We investigate the evolution of mode functions of the U(1) gauge field in the one-bubble open inflationary

¹Rigorously speaking, the proof of the absence of supercurvature modes requires knowledge of not only the equation of motion but also a Klein-Gordon norm and proper boundary conditions. The present paper fills those gaps for the analysis of the massless vector field, although our main focus will be on a massive vector field.

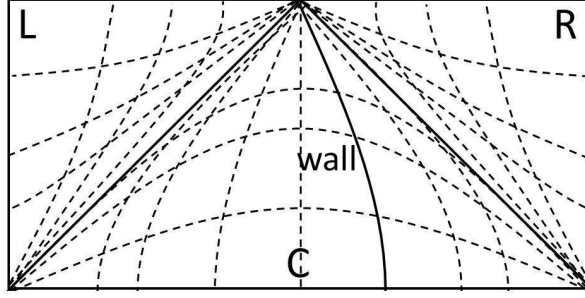


Figure 1. Penrose diagram of bubble nucleating universe.

scenario and particularly focus on whether the supercurvature modes are generated. To be specific, we introduce a real scalar field σ that governs the quantum tunneling from a false vacuum to a true vacuum and realizes inflation after the quantum tunneling, and a complex scalar field Φ that plays a major role in the coupling to the U(1) gauge field A_μ . Our action is given by

$$S = S_{\text{tun}} + S_{\text{AH}}, \quad (2.1)$$

where

$$S_{\text{tun}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_{\text{tun}}(\sigma) \right], \quad (2.2)$$

$$S_{\text{AH}} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - g^{\mu\nu} D_\mu \Phi \overline{D_\nu \Phi} - V_\Phi(|\Phi|) \right]. \quad (2.3)$$

Here the potential $V_{\text{tun}}(\sigma)$ is assumed to be the form that realizes the fast vacuum decay, $D_\mu = \partial_\mu - ieA_\mu$ is the gauge-covariant derivative, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the gauge field, and an overbar denotes the complex conjugate. Since the potential term of Φ depends only on its absolute value, this action has the local U(1) symmetry:

$$\Phi \rightarrow \Phi e^{i\alpha(x)}, \quad A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \alpha(x). \quad (2.4)$$

However, if Φ acquires a non-zero vacuum expectation value, $\langle \Phi \rangle \neq 0$, then the local U(1) symmetry is spontaneously broken. In that case, the phase degree of freedom is absorbed into the vector field and the gauge field becomes massive as it is well known as Higgs mechanism. In this paper, we consider a simple open inflation model in which the bubble nucleation can be well described by the single-field Coleman-de Luccia (CDL) instanton [24, 25] on the exact de Sitter spacetime with the Hubble parameter H . Hence we assume that the tunneling transition can be described by a Euclidean $O(4)$ -symmetric bounce solution on a Euclidean de Sitter geometry.

The Euclidean geometry can then be well described by the Euclidean de Sitter metric:

$$ds^2 = a_E^2(\eta_E) \left[d\eta_E^2 + dr_E^2 + \sin^2 r_E \omega_{ab} d\theta^a d\theta^b \right], \quad (2.5)$$

where $-\infty \leq \eta_E \leq +\infty$, $0 \leq r_E \leq \pi$, $a_E(\eta_E) = 1/H \cosh \eta_E$, and $\omega_{ab} = \text{diag}(1, \sin^2 \theta)$ denotes the metric on the unit 2-sphere. The background geometry in the Lorentzian regime is obtained by analytic continuation of the bounce solution. The coordinates in the Lorentzian regime are

$$\eta_E = \eta = -\eta_R - \frac{\pi}{2}i = \eta_L + \frac{\pi}{2}i, \quad (2.6)$$

$$r_E = -ir + \frac{\pi}{2} = -ir_R = -ir_L, \quad (2.7)$$

$$a_E = a = ia_R = ia_L. \quad (2.8)$$

Each set of these coordinates covers one of three distinct parts of the Lorentzian de Sitter spacetime, called regions-R, L, and C. Hereafter, we suppress the subscript C because we mainly work in the region-C. The Penrose diagram for this open FLRW universe is presented in Fig. 1. As seen in Fig. 1, the surfaces which respect the maximal symmetry in the region-R and L, i.e. $\eta_{R,L} = \text{const}$ hypersurfaces are not Cauchy surfaces of the whole spacetime, and hence they are not appropriate to normalize mode functions (see e.g. [26, 27]). In the region-C, however, $r = \text{const}$ hypersurfaces behave as Cauchy surfaces. Therefore, we need to find the reduced action and properly construct the Klein-Gordon (KG) norm on a Cauchy surface in the region-C. The analytic continuation of eq. (2.5) to the region-C is given by

$$ds^2 = a^2(\eta) \bar{g}_{\mu\nu} dx^\mu dx^\nu = a^2(\eta) \left[d\eta^2 - dr^2 + \cosh^2 r \omega_{ab} d\theta^a d\theta^b \right], \quad (2.9)$$

where $a(\eta) = 1/H \cosh \eta$. Note that in the region-C, $\eta = \text{const.}$ hypersurfaces are no longer spacelike, and r instead of η plays the role of a time coordinate there.

3 Reduced action and Klein-Gordon norm

To describe the Euclidean vacuum state, we need a complete set of mode functions, which should be properly normalized on a Cauchy surface. In order to determine whether supercurvature modes of the U(1) gauge field exist or not, we thus have to construct the KG norm on a Cauchy surface and to check if the modes can be properly normalized. In this section, we discuss the quantization of the U(1) gauge field in the open chart of the de Sitter spacetime and derive the KG norm on a Cauchy surface. Since we are interested only in the gauge field, we hereafter neglect the quantum fluctuations of $\varphi \equiv |\Phi|$ and treat it as a non-vanishing constant value by assuming that the mass squared $V_\Phi''(|\Phi|)$ around the potential minimum is large enough. Hence, it is convenient to decompose the complex scalar field Φ into its absolute value and phase as $\Phi(x) = \varphi e^{i\Theta(x)}$ with $\varphi = \text{const.}$ Based on the gauge transformation property, eq. (2.4), one can construct a gauge-invariant variable. In the case of the nonvanishing coupling constant, $e \neq 0$, one possible choice of such variable is

$$\mathcal{A}_\mu \equiv A_\mu - \frac{1}{e} \partial_\mu \Theta. \quad (3.1)$$

This is chosen as an appropriate variable in the unitary gauge : $\Theta = 0$. With these assumptions, the relevant part of the action (2.3) can be written in terms of the gauge-invariant variable:

$$S_{\text{eff}} = - \int d^4x \sqrt{-\bar{g}} \left[\frac{1}{4} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} (\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu) (\partial_\alpha \mathcal{A}_\beta - \partial_\beta \mathcal{A}_\alpha) + a^2 m_A^2 \bar{g}^{\mu\nu} \mathcal{A}_\mu \mathcal{A}_\nu \right], \quad (3.2)$$

where $\bar{g}_{\mu\nu}$ is the conformally transformed metric defined in eq. (2.9) and we have introduced $m_A = e\varphi$ to denote the effective mass of the gauge field.

As we mentioned above, we need to work in the region-C, where the background configuration is spatially inhomogeneous. Hence we should expand perturbations in a way that respects the symmetry of the 2-sphere rather than that of the 3-hyperboloid on which harmonics of various types are defined (see Appendix B). To rewrite the action (3.2) in terms of the $(1+1+2)$ decomposition, let us decompose A_μ into the variables $\{A_\eta, A_r, A_a\}$ with $a = \theta, \phi$. Note that A_η and A_r behave as the even parity modes with respect to the two-dimensional rotation. Since A_a behaves as 2-vector, we can further decompose it into the even and odd parity parts as

$$A_a = A_{:a}^{(\text{e})} + \epsilon_a{}^b A_{:b}^{(\text{o})}, \quad (3.3)$$

where we have introduced the colon $(:)$ as the covariant derivative with respect to the unit 2-sphere metric ω_{ab} , and $\epsilon^a{}_b$ is the unit anti-symmetric tensor on the unit 2-sphere, which are defined in eqs. (A.10), (B.19), respectively.

Before constructing the reduced action, let us consider the boundary conditions at $\eta \rightarrow \pm\infty$ for the gauge-invariant variables. Since the boundary of the open slice of the de Sitter spacetime, that is $\eta \rightarrow \pm\infty$, is regular, we can impose the condition that scalar gauge-invariant quantities such as $F_{\mu\nu} F^{\mu\nu}$ be all regular at $\eta \rightarrow \pm\infty$. In the case of the nonvanishing coupling constant, we can construct another gauge-invariant quantity \mathcal{A}_μ as defined in (3.1). We can then impose the condition that the tetrad components of the gauge-invariant vector, $\mathcal{A}_\mu e^\mu_{(\alpha)} = (\mathcal{A}_\eta/a, \mathcal{A}_r/a, \mathcal{A}_a/a)$, be regular at $\eta \rightarrow \pm\infty$. Here, $\{e^\mu_{(\alpha)}\}$ is a tetrad basis of the de Sitter spacetime. In consequence, $\mathcal{A}_\eta, \mathcal{A}_r$, and \mathcal{A}_a have to decay as fast as (or faster than) $e^{-|\eta|}$ at $\eta \rightarrow \pm\infty$.

Hereafter, using the U(1) gauge degree of freedom, we adopt the unitary gauge : $\Theta = 0$. The gauge-invariant vector \mathcal{A}_μ then reduces to the original gauge field A_μ .

We now expand the perturbations in terms of the spherical harmonics as

$$A_\eta(\eta, r, \Omega) = \sum_{\ell m} A_\eta^{\ell m}(\eta, r) Y_{\ell m}(\Omega), \quad A_r(\eta, r, \Omega) = \sum_{\ell m} A_r^{\ell m}(\eta, r) Y_{\ell m}(\Omega), \quad (3.4)$$

$$A^{(\lambda)}(\eta, r, \Omega) = \sum_{\ell m} A^{(\lambda)\ell m}(\eta, r) Y_{\ell m}(\Omega), \quad (3.5)$$

where $\lambda = \text{e}$ and o . It is straightforward to express the action in terms of the coefficients of the spherical harmonic expansion. We then find that the action can be decomposed into the even and odd parity parts as

$$S_{\text{eff}} = S^{(\text{e})} + S^{(\text{o})}, \quad (3.6)$$

where

$$S^{(e)} = \frac{1}{2} \sum_{\ell m} \int dr d\eta \left\{ \cosh^2 r \left(\partial_r A_\eta^{\ell m} - \partial_\eta A_r^{\ell m} \right)^2 + a^2 m_A^2 \cosh^2 r \left[\left(A_r^{\ell m} \right)^2 - \left(A_\eta^{\ell m} \right)^2 \right] \right. \\ \left. + \ell(\ell+1) \left[\left(\partial_r A^{(e)\ell m} - A_r^{\ell m} \right)^2 - \left(\partial_\eta A^{(e)\ell m} - A_\eta^{\ell m} \right)^2 - a^2 m_A^2 \left(A^{(e)\ell m} \right)^2 \right] \right\}, \quad (3.7)$$

is the action for the even parity modes, and

$$S^{(o)} = \frac{1}{2} \sum_{\ell m} \ell(\ell+1) \int dr d\eta \left\{ \left(\partial_r A^{(o)\ell m} \right)^2 - \left(\partial_\eta A^{(o)\ell m} \right)^2 - \left(\frac{\ell(\ell+1)}{\cosh^2 r} + a^2 m_A^2 \right) \left(A^{(o)\ell m} \right)^2 \right\}, \quad (3.8)$$

is the action for the odd parity modes.

Since the U(1) gauge field contains an auxiliary variable, or non-dynamical degree of freedom, we need to remove it to find the appropriate KG norm that contains only the physical degrees of freedom. In the region-C, A_r rather than A_η behaves as the auxiliary variable which does not have the time kinetic term. Varying the action with respect to A_r , we have the constraint equation, which is given by

$$\hat{\mathcal{O}} A_r^{\ell m} \equiv \left[-\partial_\eta^2 + m_A^2 a^2 + \frac{\ell(\ell+1)}{\cosh^2 r} \right] A_r^{\ell m} = \frac{\ell(\ell+1)}{\cosh^2 r} \partial_r A^{(e)\ell m} - \partial_\eta \partial_r A_\eta^{\ell m}. \quad (3.9)$$

The even parity modes contain two degrees of freedom. One of them is the 3-dimensional scalar mode, and the other is the 3-dimensional even parity vector mode. In order to decompose the action properly we introduce the even parity vector-type variable by

$$V^{(e)\ell m} \equiv A^{(e)\ell m} + \hat{\mathcal{K}}^{-1} \partial_\eta A_\eta^{\ell m}, \quad (3.10)$$

where $\hat{\mathcal{K}}$ is a derivative operator given by

$$\hat{\mathcal{K}} = -\partial_\eta^2 + m_A^2 a^2. \quad (3.11)$$

We then switch $\{A_\eta^{\ell m}, A^{(e)\ell m}\}$ to $\{A_\eta^{\ell m}, V^{(e)\ell m}\}$ as dynamical degrees of freedom. In terms of the new set of variables, namely $\{A_\eta^{\ell m}, V^{(e)\ell m}\}$, the constraint equation (3.9) can be rewritten as

$$A_r^{\ell m} = \hat{\mathcal{O}}^{-1} \frac{\ell(\ell+1)}{\cosh^2 r} \partial_r V^{(e)\ell m} - \hat{\mathcal{K}}^{-1} \partial_\eta \partial_r A_\eta^{\ell m}. \quad (3.12)$$

Note that we need to specify a boundary condition to properly define $\hat{\mathcal{K}}^{-1} \partial_\eta A_\eta^{\ell m}$ in eq. (3.10) since it contains an inverse operator. Different boundary conditions would lead to different prescriptions for the decomposition of $A^{(e)\ell m}$. Note that the boundary condition for $\hat{\mathcal{K}}^{-1} \partial_\eta A_\eta^{\ell m}$ must be consistent with the boundary condition for the source $\partial_\eta A_\eta^{\ell m}$ but otherwise can be specified arbitrarily for our convenience. We have already imposed the boundary condition that $A_\eta^{\ell m}, A_r^{\ell m}, A^{(e)\ell m}$ decay as fast as (or faster than) $e^{-|\eta|}$ at $\eta \rightarrow \pm\infty$. In particular this boundary condition for $A_\eta^{\ell m}$ makes it possible for us to impose the boundary condition that $\hat{\mathcal{K}}^{-1} \partial_\eta A_\eta^{\ell m}$ also decay as fast as (or faster than) $\propto e^{-|\eta|}$ at $\eta \rightarrow \pm\infty$. The boundary condition for $A^{(e)\ell m}$ then implies that $V^{(e)\ell m}$ also decays as fast as (or faster than) $\propto e^{-|\eta|}$ at $\eta \rightarrow \pm\infty$.

Substituting eqs. (3.10) and (3.12) into eq. (3.7), after lengthly calculation, we obtain the reduced action only for the dynamical degrees of freedom. The resultant reduced action is given by

$$S^{(e)} = S_s^{(e)} + S_v^{(e)}, \quad (3.13)$$

where

$$S_s^{(e)} = \frac{1}{2} \sum_{\ell m} \int dr d\eta \left\{ \cosh^2 r \left(\partial_r A_\eta^{\ell m} \right) \left[1 + \partial_\eta \hat{\mathcal{K}}^{-1} \partial_\eta \right] \left(\partial_r A_\eta^{\ell m} \right) - \ell(\ell+1) A_\eta^{\ell m} \left[1 + \partial_\eta \hat{\mathcal{K}}^{-1} \partial_\eta \right] A_\eta^{\ell m} - m_A^2 a^2 \cosh^2 r \left(A_\eta^{\ell m} \right)^2 \right\}, \quad (3.14)$$

for the scalar mode, and

$$S_v^{(e)} = \frac{1}{2} \sum_{\ell m} \ell(\ell+1) \int dr d\eta \left\{ \left(\partial_r V^{(e)\ell m} \right) \left[1 - \hat{\mathcal{O}}^{-1} \frac{\ell(\ell+1)}{\cosh^2 r} \right] \left(\partial_r V^{(e)\ell m} \right) - V^{(e)\ell m} \hat{\mathcal{K}} V^{(e)\ell m} \right\}, \quad (3.15)$$

for the vector mode. Here, we have used the boundary conditions for $A_\eta^{\ell m}$ and $V^{(e)\ell m}$ to show that some boundary terms, such as those including the interaction between $A_\eta^{\ell m}$ and $V^{(e)\ell m}$, vanish. Hence the scalar and vector modes are completely decoupled in the action for the even parity mode.

We can now define the KG norm by using the reduced actions obtained above, following and extending [36].² To quantize the system of the U(1) gauge field, we promote the physical degrees of freedom \mathbf{A} to operators $\hat{\mathbf{A}}$, and expand $\hat{\mathbf{A}}$ by mode functions $\{\mathbf{A}_\mathcal{N}, \overline{\mathbf{A}}_\mathcal{N}\}$ as

$$\hat{\mathbf{A}}(x) = \sum_{\mathcal{N}} \left[\hat{a}_\mathcal{N} \mathbf{A}_\mathcal{N}(x) + \hat{a}_\mathcal{N}^\dagger \overline{\mathbf{A}}_\mathcal{N}(x) \right], \quad (3.16)$$

where $\hat{a}_\mathcal{N}$ and $\hat{a}_\mathcal{N}^\dagger$ are the annihilation and creation operators, respectively, that satisfy the commutation relation, $[\hat{a}_\mathcal{N}, \hat{a}_\mathcal{M}^\dagger] = \delta_{\mathcal{N}\mathcal{M}}$. The quantum fluctuations of the field are described by the vacuum state $|0\rangle$ such that $\hat{a}_\mathcal{N}|0\rangle = 0$ for any \mathcal{N} . We note that $\{\mathbf{A}_\mathcal{N}, \overline{\mathbf{A}}_\mathcal{N}\}$ should form a complete set of linearly independent solutions of the equation of motion. With these variables, we define the KG norms as

$$(\mathbf{A}_\mathcal{N}, \mathbf{A}_\mathcal{M})_{\text{KG}}^{(s)} = -i \cosh^2 r \int d\eta d\Omega \left\{ A_{\eta,\mathcal{N}} \left[1 + \partial_\eta \hat{\mathcal{K}}^{-1} \partial_\eta \right] \partial_r \overline{A_{\eta,\mathcal{M}}} - \left[1 + \partial_\eta \hat{\mathcal{K}}^{-1} \partial_\eta \right] \partial_r A_{\eta,\mathcal{N}} \overline{A_{\eta,\mathcal{M}}} \right\}, \quad (3.17)$$

for the even parity scalar modes,

$$(\mathbf{A}_\mathcal{N}, \mathbf{A}_\mathcal{M})_{\text{KG}}^{(v)} = -i \ell(\ell+1) \int d\eta d\Omega \left\{ V_\mathcal{N}^{(e)} \left[1 - \hat{\mathcal{O}}^{-1} \frac{\ell(\ell+1)}{\cosh^2 r} \right] \partial_r \overline{V_\mathcal{M}^{(e)}} - \left[1 - \hat{\mathcal{O}}^{-1} \frac{\ell(\ell+1)}{\cosh^2 r} \right] \partial_r V_\mathcal{N}^{(e)} \overline{V_\mathcal{M}^{(e)}} \right\}, \quad (3.18)$$

²As we shall see below, an equivalent method to define the KG norm is provided by a general formula derived in Appendix C. An advantage of this alternative method is that it can be applied without eliminating auxiliary fields in the action. The result is of course the same, as far as the boundary conditions specified above are imposed.

for the even parity vector modes, and

$$(\mathbf{A}_{\mathcal{N}}, \mathbf{A}_{\mathcal{M}})_{\text{KG}}^{(\text{o})} = -i\ell(\ell+1) \int d\eta d\Omega \left\{ A_{\mathcal{N}}^{(\text{o})} \partial_r \overline{A_{\mathcal{M}}^{(\text{o})}} - \left(\partial_r A_{\mathcal{N}}^{(\text{o})} \right) \overline{A_{\mathcal{M}}^{(\text{o})}} \right\}, \quad (3.19)$$

for the odd parity modes. With the KG norm defined above, all modes should be properly normalized on a Cauchy surface as

$$(\mathbf{A}_{\mathcal{N}}, \mathbf{A}_{\mathcal{M}})_{\text{KG}}^{(\lambda)} = \delta_{\mathcal{N}\mathcal{M}}, \quad (3.20)$$

with $\lambda = \text{s}, \text{v}, \text{o}$. Once $A_{\eta}^{\ell m}$, $V^{(\text{e})\ell m}$ and $A_{\eta}^{(\text{o})}$ are properly evaluated by solving the equation of motion, we can calculate the KG norm through eqs. (3.17)-(3.20).

In some cases it is convenient to rewrite the reduced action and the KG norm in terms of auxiliary fields. Introducing the new auxiliary fields, $\mathcal{S}_r^{\ell m}$ and $V_r^{\ell m}$, which obey the constraint equations:

$$\hat{\mathcal{K}} \mathcal{S}_r^{\ell m} = -\partial_{\eta} \partial_r A_{\eta}^{\ell m}, \quad \hat{\mathcal{O}} V_r^{\ell m} = \frac{\ell(\ell+1)}{\cosh^2 r} \partial_r V^{(\text{e})\ell m}, \quad (3.21)$$

$A_r^{\ell m}$ in eq. (3.12) can be reduced to

$$A_r^{\ell m} = V_r^{\ell m} + \mathcal{S}_r^{\ell m}, \quad (3.22)$$

and we can use $\mathcal{S}_r^{\ell m}$ and $V_r^{\ell m}$ as the auxiliary fields for the scalar and vector modes rather than $A_r^{\ell m}$. With these variables, the reduced actions for the scalar- and vector-modes are rewritten as

$$S_{\text{sca}}^{(\text{e})} = \frac{1}{2} \sum_{\ell m} \int dr d\eta \left\{ \cosh^2 r \left(\partial_r A_{\eta}^{\ell m} - \partial_{\eta} \mathcal{S}_r^{\ell m} \right)^2 + m_A^2 a^2 \cosh^2 r \left(\mathcal{S}_r^{\ell m} \right)^2 - \ell(\ell+1) A_{\eta}^{\ell m} \left[1 + \partial_{\eta} \hat{\mathcal{K}}^{-1} \partial_{\eta} \right] A_{\eta}^{\ell m} - m_A^2 a^2 \cosh^2 r \left(A_{\eta}^{\ell m} \right)^2 \right\}, \quad (3.23)$$

$$S_{\text{vec}}^{(\text{e})} = \frac{1}{2} \sum_{\ell m} \int dr d\eta \left\{ \ell(\ell+1) \left(\partial_r V^{(\text{e})\ell m} - V_r^{\ell m} \right)^2 + \cosh^2 r V_r^{\ell m} \hat{\mathcal{K}} V_r^{\ell m} - \ell(\ell+1) V^{(\text{e})\ell m} \hat{\mathcal{K}} V^{(\text{e})\ell m} \right\}. \quad (3.24)$$

Following the same step as discussed in Appendix C, we can define the KG norm in terms of the auxiliary fields as

$$(\mathbf{A}_{\mathcal{N}}, \mathbf{A}_{\mathcal{M}})_{\text{KG}}^{(\text{s})} = -i \cosh^2 r \int d\eta d\Omega \left\{ A_{\eta, \mathcal{N}} \left(\partial_r \overline{A_{\eta, \mathcal{M}}} - \partial_{\eta} \overline{\mathcal{S}_{r, \mathcal{M}}} \right) - \left(\partial_r A_{\eta, \mathcal{N}} - \partial_{\eta} \mathcal{S}_{r, \mathcal{N}} \right) \overline{A_{\eta, \mathcal{M}}} \right\}, \quad (3.25)$$

$$(\mathbf{A}_{\mathcal{N}}, \mathbf{A}_{\mathcal{M}})_{\text{KG}}^{(\text{v})} = -i\ell(\ell+1) \int d\eta d\Omega \left\{ V_{\mathcal{N}}^{(\text{e})} \left(\partial_r \overline{V_{\mathcal{M}}^{(\text{e})}} - \overline{V_{r, \mathcal{M}}} \right) - \left(\partial_r V_{\mathcal{N}}^{(\text{e})} - V_{r, \mathcal{N}} \right) \overline{V_{\mathcal{M}}^{(\text{e})}} \right\}, \quad (3.26)$$

for the even parity scalar and vector modes, respectively, where the auxiliary fields $\mathcal{S}_r^{\ell m}$ and $V_r^{\ell m}$ are determined by the constraint equation (3.21). It is easy to see that these expressions for the KG norm are equivalent to (3.17)-(3.20).

4 Mode functions

In this section we construct a complete set of mode functions. Since odd and even parity sectors are decoupled, we shall investigate each sector separately.

4.1 Odd parity modes

First we consider the odd parity sector. The odd parity sector contains one dynamical degree of freedom, which corresponds to the odd parity part of a 3-dimensional transverse vector. Let us construct a set of positive frequency functions corresponding to the variable $A^{(o)}$. Varying the action (3.8) with respect to $A^{(o)}$, we obtain

$$\left[\partial_r^2 - \partial_\eta^2 + \frac{\ell(\ell+1)}{\cosh^2 r} + m_A^2 a^2 \right] A^{(o)\ell m} = 0 \quad (4.1)$$

In order to solve this equation, we expand $A^{(o)}$ as

$$A^{(o)\ell m}(\eta, r) = \sum_p v_p^{(o)}(\eta) \left(\frac{1}{\sqrt{\ell(\ell+1)}} \cosh r f^{p\ell}(r) \right), \quad (4.2)$$

where the “summation” on the r.h.s. should be understood as the integral over continuum modes ($p^2 > 0$) plus the summation over discrete modes ($p^2 < 0$), if any. Here, we have fixed the coefficient in front of $f^{p\ell}(r)$ so that the expression inside the parenthesis, when multiplied by $Y_{\ell m; b} \epsilon_a^b$, corresponds to the odd-mode vector-type harmonic function on a unit 3-hyperboloid analytically-continued to the region-C (see eqs. (B.25) and (2.7)). Appendix B summarizes the characteristics of the scalar- and vector-type harmonic functions in the open universe. The equation for $f^{p\ell}$ is given by

$$\left[-\frac{1}{\cosh^2 r} \frac{d}{dr} \left(\cosh^2 r \frac{d}{dr} \right) - \frac{\ell(\ell+1)}{\cosh^2 r} \right] f^{p\ell}(r) = (p^2 + 1) f^{p\ell}(r). \quad (4.3)$$

Adopting the Euclidean vacuum state as a natural choice after quantum tunneling, we impose that the positive frequency functions are regular at $r = 0$ (see eq. (B.5)). We then have the explicit expression for $f^{p\ell}$ as

$$f^{p\ell}(r) = \sqrt{\frac{\Gamma(ip + \ell + 1)\Gamma(-ip + \ell + 1)}{i\Gamma(ip)\Gamma(-ip) \cosh r}} P_{ip - \frac{1}{2}}^{-\ell - \frac{1}{2}}(i \sinh r), \quad (4.4)$$

where P_ν^μ is the associated Legendre function of the first kind, and we have fixed the normalization constant so that the analytic continuation of $Y^{p\ell m}(r, \Omega) \equiv f^{p\ell}(r) Y_{\ell m}(\Omega)$ to the region-R or L behaves as a harmonic function properly normalized on a unit 3-hyperboloid (see Appendix B).

In this expression, $v_p^{(o)}$ is an eigenfunction of the operator $\hat{\mathcal{K}}$ with the eigenvalue p^2 , that is, $v_p^{(o)}$ satisfies

$$\hat{\mathcal{K}} v_p^{(o)} = \left[-\frac{d^2}{d\eta^2} + m_A^2 a^2 \right] v_p^{(o)} = p^2 v_p^{(o)}. \quad (4.5)$$

The boundary condition for $A^{(o)\ell m}$ is that $v_p^{(o)}$ should decay as fast as (or faster than) $e^{-|\eta|}$ at $\eta \rightarrow \pm\infty$. Since the effective potential $m_A^2 a^2$ is clearly positive definite, this in particular

implies that there is no solution with negative p^2 . It is thus concluded that there is no supercurvature mode ($p^2 < 0$ mode) in the odd parity sector.

To find the two independent solutions for $v_p^{(o)}$ with $p^2 > 0$, it is useful to introduce the two normalized orthogonal solutions $\varpi_{\pm,p}$ which satisfy

$$\hat{\mathcal{K}}\varpi_{\pm,p} = \left[-\frac{d^2}{d\eta^2} + m_A^2 a^2 \right] \varpi_{\pm,p} = p^2 \varpi_{\pm,p}. \quad (4.6)$$

Since $a = 1/H \cosh \eta$, it is easy to solve this equation, and the general solution is

$$\varpi_{\pm,p} = C_{1,p}^{\pm} P_{\nu'}^{ip}(-\tanh \eta) + C_{2,p}^{\pm} P_{\nu'}^{-ip}(-\tanh \eta), \quad (4.7)$$

where $C_{1,p}^{\pm}$ and $C_{2,p}^{\pm}$ are constants,

$$\nu' = \sqrt{\frac{9}{4} - \frac{M_{\text{eff}}^2}{H^2}} - \frac{1}{2}, \quad M_{\text{eff}}^2 = m^2 + 2H^2. \quad (4.8)$$

To construct the independent solutions, let us consider the scattering problem for $\varpi_{\pm,p}$. Since the solutions asymptotically approach linear combinations of the plane waves $e^{\pm ip\eta}$ as $\eta \rightarrow \pm\infty$, the equation (4.6) describes incident plane waves interacting with the effective potential $m_A^2 a^2$ and producing reflected and transmitted waves to $\eta \rightarrow \pm\infty$ and $\eta \rightarrow \mp\infty$, respectively. We then take the two independent solutions having the following asymptotic behaviors:

$$\varpi_{+,p} \rightarrow \begin{cases} \rho_{+,p} e^{+ip\eta} + e^{-ip\eta} & : \eta \rightarrow +\infty \\ \sigma_{+,p} e^{-ip\eta} & : \eta \rightarrow -\infty \end{cases}, \quad (4.9)$$

$$\varpi_{-,p} \rightarrow \begin{cases} \rho_{-,p} e^{-ip\eta} + e^{+ip\eta} & : \eta \rightarrow -\infty \\ \sigma_{-,p} e^{+ip\eta} & : \eta \rightarrow +\infty \end{cases}. \quad (4.10)$$

The reflection and transmission coefficients satisfy the following Wronskian relations [29]:

$$|\rho_{\pm,p}|^2 + |\sigma_{\pm,p}|^2 = 1, \quad \sigma_{+,p} = \sigma_{-,p}, \quad \sigma_{+,p} \overline{\rho_{-,p}} + \overline{\sigma_{-,p}} \rho_{+,p} = 0. \quad (4.11)$$

These solutions are shown to be orthogonal,

$$\int_{-\infty}^{\infty} d\eta w_{\sigma,p} \overline{w_{\sigma',p'}} = 2\pi \delta_{\sigma\sigma'} \delta_D(p - p'). \quad (4.12)$$

Comparing the asymptotic behavior of the exact solution (4.7) and eqs. (4.9)-(4.11), we find the corresponding coefficients as

$$C_{1,p}^+ = \frac{\Gamma(-ip - \nu') \Gamma(1 - ip + \nu')}{\Gamma(-ip)}, \quad C_{2,p}^+ = 0, \quad (4.13)$$

$$C_{1,p}^- = \frac{\sin(\pi\nu')}{\pi} \Gamma(1 - ip) \Gamma(-ip - \nu') \Gamma(1 - ip + \nu'), \quad C_{2,p}^- = \Gamma(1 - ip). \quad (4.14)$$

The two independent solutions for the odd parity modes are expressed in terms of these solutions, namely $v_{\pm,p}^{(o)} \propto \varpi_{\pm,p}$. If η were the time variable then either one of $e^{\pm ip\eta}$ would be chosen by a boundary condition to specify a quantum state of the system (e.g. the Bunch-Davies vacuum). However, in the present situation, it is r that is the time variable and thus

a quantum state of the system is chosen by imposing a boundary condition on the function of r as in eq. (4.4). Hence both of $e^{\pm ip\eta}$ should be treated as independent mode functions and are needed for the construction of a complete set of mode functions. In order to quantize the perturbations, we introduce a variable defined by

$$A_{\sigma p \ell m}^{(o)}(\eta, r, \Omega) = N_p^{(o)} \varpi_{\sigma, p}(\eta) \left(\frac{1}{\sqrt{\ell(\ell+1)}} \cosh r f^{p\ell}(r) Y_{\ell m}(\Omega) \right), \quad (4.15)$$

where $N_p^{(o)}$ is a normalization constant. Recalling that we have already proven the absence of supercurvature modes in the odd parity sector (see eq. (4.5)) and substituting this into eq. (3.19), we require

$$\begin{aligned} (\mathbf{A}_{\sigma p \ell m}, \mathbf{A}_{\sigma' p' \ell' m'})_{\text{KG}}^{(o)} &= 4p \sinh(\pi p) \left(N_p^{(o)} \right)^2 \delta_{\sigma\sigma'} \delta_{\text{D}}(p - p') \delta_{\ell\ell'} \delta_{mm'} \\ &= \delta_{\sigma\sigma'} \delta_{\text{D}}(p - p') \delta_{\ell\ell'} \delta_{mm'}, \end{aligned} \quad (4.16)$$

where we have used the orthogonality condition for $\varpi_{\sigma, p}$ (see eq. (4.12)). Hence the KG normalization condition implies that the normalization constant is given by

$$N_p^{(o)} = \frac{1}{2\sqrt{p \sinh(\pi p)}}. \quad (4.17)$$

In summary, we have shown that there is no supercurvature mode in the odd parity sector and that continuous odd parity modes (with $p^2 > 0$) are given by (4.15) with (4.17), (4.7)-(4.8) and (4.13)-(4.14).

4.2 Even parity modes

In this subsection, we construct a complete set of mode functions in the even parity sector. The equations for $A_\eta^{\ell m}$ and $V^{(e)\ell m}$ are given by

$$\left[-\frac{1}{\cosh^2 r} \partial_r (\cosh^2 r \partial_r) - \frac{\ell(\ell+1)}{\cosh^2 r} \right] (1 + \partial_\eta \hat{\mathcal{K}}^{-1} \partial_\eta) A_\eta^{\ell m} = m_A^2 a^2 A_\eta^{\ell m}, \quad (4.18)$$

for the 3-dimensional scalar modes, and

$$\hat{\mathcal{K}} V^{(e)\ell m} = -\partial_r \left[\left(1 - \hat{\mathcal{O}}^{-1} \frac{\ell(\ell+1)}{\cosh^2 r} \right) \partial_r V^{(e)\ell m} \right], \quad (4.19)$$

for the 3-dimensional vector modes. Assuming $m_A^2 \neq 0$, we then expand $A_\eta^{\ell m}$ and $V^{(e)\ell m}$ as

$$A_\eta^{\ell m}(\eta, r) = \sum_p \frac{\chi_p(\eta)}{a(\eta)} f^{p\ell}(r), \quad (4.20)$$

$$V^{(e)\ell m}(\eta, r) = \sum_p v_p^{(e)}(\eta) \left[\frac{1}{\sqrt{\ell(\ell+1)p}} \frac{d}{dr} (\cosh r f^{p\ell}(r)) \right], \quad (4.21)$$

where the coefficients have been fixed so that the analytic continuations of these functions to region-R or L corresponds to the scalar- and vector-type harmonic functions defined in

Appendix B. Thus, the equations for χ_p and $v_p^{(e)}$ can be reduced to

$$\left[-\frac{d^2}{d\eta^2} + \left(m_A^2 a^2 + a \frac{d^2}{d\eta^2} \left(\frac{1}{a} \right) - 1 \right) \right] \chi_p = \left[-\frac{d^2}{d\eta^2} + m_A^2 a^2 \right] \chi_p = p^2 \chi_p, \quad (4.22)$$

$$\left[-\frac{d^2}{d\eta^2} + m_A^2 a^2 \right] v_p^{(e)} = p^2 v_p^{(e)}. \quad (4.23)$$

These equations for χ_p and $v_p^{(e)}$ are exactly the same as that for $\varpi_{\pm,p}$ investigated in the previous subsection (see eqs. (4.6)-(4.14)). The boundary conditions for $A_\eta^{\ell m}$ and $V^{(e)\ell m}$ imply that χ_p and $v_p^{(e)}$ decay as fast as (or faster than) $e^{-2|\eta|}$ and $e^{-|\eta|}$, respectively, at $\eta \rightarrow \pm\infty$. It is thus concluded that there is no supercurvature mode in the even parity sector for the same reason as in the odd parity sector, i.e. because of the positivity of the effective potential $m_A^2 a^2$.

It is also straightforward to repeat the same procedure as in the previous subsection to find a complete set of continuous ($p^2 > 0$) mode functions in the even parity sector. Mode functions are simply expressed in terms of $\varpi_{\pm,p}$. To quantize the perturbations, we introduce variables:

$$A_{\eta, \sigma p \ell m}(\eta, r, \Omega) = N_p^{(s)} \frac{\varpi_{\sigma,p}(\eta)}{a(\eta)} f^{p\ell}(r) Y_{\ell m}(\Omega), \quad (4.24)$$

$$V_{\sigma p \ell m}^{(e)}(\eta, r, \Omega) = N_p^{(v)} \varpi_{\sigma,p}(\eta) \left[\frac{1}{\sqrt{\ell(\ell+1)p}} \frac{d}{dr} \left(\cosh r f^{p\ell}(r) \right) Y_{\ell m}(\Omega) \right], \quad (4.25)$$

where $N_p^{(s)}$ and $N_p^{(v)}$ are normalization constants to be determined for the scalar and vector modes, respectively. We can determine the normalization constants by using the KG norms. Substituting eqs. (4.24) and (4.25) into the KG norm for the even parity modes defined in eqs. (3.17) and (3.18), we have

$$(\mathbf{A}_{\sigma p \ell m}, \mathbf{A}_{\sigma' p' \ell' m'})_{\text{KG}}^{(s)} = m_A^2 \frac{4p \sinh(\pi p)}{p^2 + 1} \left(N_p^{(s)} \right)^2 \delta_{\sigma\sigma'} \delta_D(p - p') \delta_{\ell\ell'} \delta_{mm'}, \quad (4.26)$$

for the 3-dimensional scalar mode, and

$$(\mathbf{A}_{\sigma p \ell m}, \mathbf{A}_{\sigma' p' \ell' m'})_{\text{KG}}^{(v)} = 4p \sinh(\pi p) \left(N_p^{(v)} \right)^2 \delta_{\sigma\sigma'} \delta_D(p - p') \delta_{\ell\ell'} \delta_{mm'}, \quad (4.27)$$

for the 3-dimensional vector mode. When we require the normalization condition eq. (3.20), the scalar- and vector-modes are normalized respectively as

$$N_p^{(s)} = \frac{1}{2m_A} \sqrt{\frac{p^2 + 1}{p \sinh(\pi p)}}, \quad N_p^{(v)} = \frac{1}{2\sqrt{p \sinh(\pi p)}}. \quad (4.28)$$

In summary, we have found that there is no supercurvature mode in the even parity sector and that a complete set of even parity continuous mode functions is given by (4.24), (4.25) with (4.28).

5 Consistency of neutral case and decoupling limit

In the previous section, we have shown that there is no supercurvature mode for a U(1) gauge field with both gauge and conformal symmetries spontaneously broken through the Higgs mechanism, for any values of the mass of the vector field.

It has been known that a scalar field ϕ with a sufficiently light effective mass, $0 \leq m < \sqrt{2}H$, has a supercurvature mode [26], and the supercurvature mode survives the massless limit. Furthermore, the existence of the supercurvature mode is essential for the recovery of the correct massless limit of the Wightman function. On one hand, one can show that the Euclidean Wightman function in the limit $m \rightarrow 0$ contains a constant divergent contribution [26]:

$$\lim_{m \rightarrow 0} \langle 0 | \phi(x) \phi(x') | 0 \rangle = \frac{3H^4}{8\pi^2 m^2} + \mathcal{O}(m^0). \quad (5.1)$$

On the other hand, the contribution of all subcurvature modes (with $p^2 > 0$) to the Wightman function remains finite in the massless limit. This means that the set of all subcurvature modes does not form a complete set of mode functions and that something is missing. It is the supercurvature mode that is missing here. The contribution from the supercurvature mode (with $p^2 = -1$) correctly reproduces the divergent behavior of the Euclidean Wightman function shown in eq. (5.1).

From the above observation on the Wightman function of a scalar field, it is expected that the massless limit serves as a useful consistency check also for vector fields. We thus consider the massless limit of the massive vector field and see whether the correct behavior of the Wightman function can be reproduced by the contributions from subcurvature modes only, without need for any supercurvature modes.

For the system of the U(1) gauge field considered in the present paper, the massless limit is provided by the decoupling limit, i.e. the $e \rightarrow 0$ limit. As we shall see in the following, the decoupling limit appears to be rather confusing. On one hand, we have shown that the massive vector field does not have a supercurvature mode for any non-zero value of e . On the other hand, for $e = 0$, i.e. if the (would-be) Higgs field is neutral under the U(1), the system consisting of the U(1) gauge field and the phase of the complex (would-be) Higgs field is reduced to a massless vector field plus a massless scalar field. Since a massless scalar field is known to have a supercurvature mode, there appears discontinuity in the $e \rightarrow 0$ limit. We need to reconcile these two apparently contradicting results.

In this section we first reconcile the apparent contradiction between the $e \rightarrow 0$ limit of the $e \neq 0$ theory and the $e = 0$ theory (subsection 5.1). We then investigate the Wightman function in the decoupling ($e \rightarrow 0$) limit as a consistency check (subsection 5.2).

5.1 Neutral ($e = 0$) case

Before considering the decoupling ($e \rightarrow 0$) limit, let us investigate the neutral ($e = 0$) case. In this subsection, we focus only on the scalar sector since the apparent contradiction explained above is in this sector. One can begin with the action eq. (3.2) with $e = 0$ to derive the equation of motion of the scalar degree of freedom. Adopting the gauge condition $A_\eta = 0$ for convenience, we obtain the action for the phase of the (would-be) Higgs field as

$$S^{(e)} \supset \frac{1}{2} \varphi^2 \sum_{\ell m} \int dr d\eta a^2 \cosh^2 r \left\{ \left(\partial_r \Theta^{\ell m} \right)^2 - \left(\partial_\eta \Theta^{\ell m} \right)^2 - \frac{\ell(\ell+1)}{\cosh^2 r} \left(\Theta^{\ell m} \right)^2 \right\}, \quad (5.2)$$

where $\Theta^{\ell m}$ is the coefficient of the spherical harmonic expansion of the phase of the (would-be) Higgs field Θ . Expanding $\Theta^{\ell m}$ in terms of $f^{p\ell}$ (see eqs. (4.3) and (4.4)) as $\Theta^{\ell m}(\eta, r) = \sum_p \Theta_p(\eta) f^{p\ell}(r)$, we obtain the equation for Θ_p as

$$\left[\frac{1}{a^2} \frac{d}{d\eta} \left(a^2 \frac{d}{d\eta} \right) + (p^2 + 1) \right] \Theta_p = 0. \quad (5.3)$$

Since this is the same as the equation of motion for the massless scalar field, one might think that there should be a supercurvature mode at $p^2 = -1$, according to [26]. However, the solution to this equation with $p^2 = -1$ is trivial, namely $\Theta_p = \text{const}$, in the entire region- \mathcal{C} and turns out to be a gauge degree of freedom. A key observation here is that in the neutral ($e = 0$) case, the $U(1)$ gauge symmetry manifests itself as a global shift symmetry: $\Theta(x) \rightarrow \Theta(x) + \lambda$, where λ is a constant. Note that this shift symmetry must be respected by any interactions including Θ . In particular, observers or detectors interacting with Θ can probe derivatives $\partial_\mu \Theta$ but cannot probe the value of Θ itself even in principle. Hence, the constant solution with $p^2 = -1$ is not within the physical spectrum of the theory. In other words, the $p^2 = -1$ solution does not affect any correlation functions invariant under the global shift symmetry since Θ enters invariant quantities only through its derivatives.

Therefore it is concluded that there is no supercurvature mode in the neutral ($e = 0$) case. This reconciles the apparent contradiction between the $e = 0$ theory and the $e \rightarrow 0$ limit of the $e \neq 0$ theory.

5.2 Decoupling ($e \rightarrow 0$) limit

Let us now explore the massless limit of the massive $U(1)$ gauge field, i.e. the decoupling limit, $e \rightarrow 0$, of the theory with $e \neq 0$. In this subsection we compute the Wightman function of the $U(1)$ gauge field in the decoupling limit and explicitly verify that the correct behavior of the Euclidean Wightman function is reproduced by the contributions from subcurvature modes only, without need for any supercurvature modes.

The Wightman function for the massive $U(1)$ gauge field in de Sitter spacetime is previously studied in the literature [37–40]³. According to [37], the Wightman function of the massive $U(1)$ gauge field in the decoupling ($e \rightarrow 0$) limit can be written in terms of the scalar propagator as

$$\lim_{m_A \rightarrow 0} \langle 0 | A_\mu(x) A_{\mu'}(x') | 0 \rangle = \lim_{m_A \rightarrow 0} \frac{1}{m_A^2} \partial_\mu \partial_{\mu'} \Delta_{M^2}(Z(x, x')) + \mathcal{O}(m_A^0). \quad (5.4)$$

where Z denotes the de Sitter invariant distance between two points, x and x' , in de Sitter spacetime, and $\Delta_{M^2}(Z)$ denotes the propagator of the scalar field with the mass $M = \sqrt{9/4 - (\nu' + 3/2)^2}$, which is defined by

$$\Delta_{M^2}(Z) = \frac{H^2}{(4\pi)^2} \frac{\Gamma(3 + \nu') \Gamma(-\nu')}{\Gamma(2)} {}_2F_1 \left(3 + \nu', -\nu'; 2; \frac{1 + Z}{2} \right), \quad (5.5)$$

where ${}_2F_1(a, b; c; z)$ is the hypergeometric function. It should be noted that the propagator of the massive scalar field in the massless (decoupling) limit is divergent as seen in eq. (5.1). However, the divergent contribution is Z -independent and thus drops out when derivatives are acted on the propagator as in (5.4). We then take the massless limit and describe the explicit expression for the divergent contributions of the Wightman function for the $U(1)$ gauge field as

$$\lim_{m_A \rightarrow 0} \langle 0 | A_\mu(x) A_{\mu'}(x') | 0 \rangle = \frac{H^2}{(4\pi)^2 m_A^2} \left[\frac{Z - 3}{(Z - 1)^3} (\partial_\mu Z) (\partial_{\mu'} Z) - \frac{Z - 2}{(Z - 1)^2} (\partial_\mu \partial_{\mu'} Z) \right]. \quad (5.6)$$

³ In [37], the authors found that the Wightman function of a massive gauge field depends on the way how gauge is fixed. Our gauge choice corresponds to what they call the Proca theory, and in this gauge the massless limit of the Wightman function has a simple form. It seems that the behavior of the Wightman function of a massive gauge field in de Sitter spacetime is not yet fully understood [38–40].

Hereafter we neglect higher-order contributions of order $\mathcal{O}(m_A^0)$ for simplicity.

In order to compare the leading-order Wightman function in the decoupling limit with our result derived in the present paper, we rewrite eq. (5.6) in terms of the coordinates in the open chart of the de Sitter spacetime i.e. the coordinate in the region-J (η_J, r_J, Ω) . The invariant distance Z is then reduced to

$$Z(x_J, x'_J) = \frac{\cosh \eta_J \cosh \eta'_J - \cosh \zeta}{\sinh \eta_J \sinh \eta'_J}, \quad (5.7)$$

where $\cosh \zeta \equiv \cosh r_J \cosh r'_J - \sinh r_J \sinh r'_J \cos \Xi$ with $\cos \Xi$ being the directional cosine between Ω and Ω' . Substituting the invariant distance eq. (5.7) into eq. (5.6), we can easily rewrite each component of the Wightman function of the U(1) gauge field in term of the coordinate in the region-J.

We also calculate the Wightman function by using the explicit expressions for the mode functions derived in section 4. As an example, let us focus on the (η, η') -component of the Wightman function of the U(1) gauge field. Taking the massless (decoupling) limit ($\nu' \rightarrow 0$) and the analytic continuation to the region-J, we can rewrite the two independent solutions for the η -component of the U(1) gauge field (4.24) as

$$A_{\eta, +p\ell m}(\eta_J, r_J, \Omega) = \frac{1}{2m_A} \sqrt{\frac{p^2 + 1}{p \sinh(\pi p)}} \frac{1}{a_J(\eta_J)} e^{ip\eta_J - \pi p/2} f^{p\ell}(r_J) Y_{\ell m}(\Omega), \quad (5.8)$$

$$A_{\eta, -p\ell m}(\eta_J, r_J, \Omega) = \frac{1}{2m_A} \sqrt{\frac{p^2 + 1}{p \sinh(\pi p)}} \frac{\Gamma(1 - ip)}{\Gamma(1 + ip)} \frac{1}{a_J(\eta_J)} e^{-ip\eta_J + \pi p/2} f^{p\ell}(r_J) Y_{\ell m}(\Omega), \quad (5.9)$$

where $a_J(\eta_J) = -1/H \sinh \eta_J$, and we have used the following relation: $P_0^{ip}(-\tanh \eta) = e^{ip\eta_J - \pi p/2} / \Gamma(1 - ip)$. We can then calculate the (η, η') -component of the Wightman function for the U(1) gauge field as

$$\begin{aligned} \lim_{m_A \rightarrow 0} \langle 0 | A_\eta(x) A_{\eta'}(x') | 0 \rangle &= \lim_{m_A \rightarrow 0} \sum_{\sigma=\pm} \sum_{p\ell m} A_{\eta, \sigma p\ell m}(\eta_J, r_J, \Omega) \overline{A_{\eta, \sigma p\ell m}(\eta'_J, r'_J, \Omega')} \\ &= \frac{H^2 \sinh \eta_J \sinh \eta'_J}{4\pi^2 m_A^2} \int_0^\infty dp \frac{(p^2 + 1) \sin(p\zeta)}{\sinh \zeta} \left\{ \frac{1}{1 - e^{-2\pi p}} e^{-ip(\eta_J - \eta'_J)} + \frac{e^{-2\pi p}}{1 - e^{-2\pi p}} e^{+ip(\eta_J - \eta'_J)} \right\} \\ &= \frac{H^2}{8\pi^2 m_A^2} \sinh \eta_J \sinh \eta'_J \frac{2 + \cosh^2 \zeta - 3 \cosh \zeta \cosh(\eta_J - \eta'_J)}{(\cosh \zeta - \cosh(\eta_J - \eta'_J))^3}, \end{aligned} \quad (5.10)$$

where we have used the completeness relation for the scalar harmonics, $Y^{p\ell m}(r_J, \Omega) \equiv f^{p\ell}(r_J) Y_{\ell m}(\Omega)$, which is given by

$$\sum_{\ell m} Y^{p\ell m}(r_J, \Omega) \overline{Y^{p\ell m}(r'_J, \Omega')} = \frac{p \sin(p\zeta)}{2\pi^2 \sinh \zeta}. \quad (5.11)$$

One can easily compare the resultant Wightman function eq. (5.10) with one obtained by substituting eq. (5.7) into (5.6) and find that these leading-order expressions exactly coincide. This confirms that the (η, η') -component of the Wightman function of the U(1) gauge field in the decoupling limit is correctly reproduced by the contribution from subcurvature modes only, without need for supercurvature modes. Following the same step as the (η, η') -component, we can verify the consistency between eq. (5.6) and our results for the other components.

6 Summary

In this paper, we have investigated the Euclidean vacuum mode functions of a massive vector field in the spatially open chart of de Sitter spacetime. In order to clarify whether supercurvature modes exist, we have studied the $U(1)$ gauge field with gauge and conformal symmetries spontaneously broken through the Higgs mechanism. We have found that there is no supercurvature mode for both the even and odd parity sectors. This implies that it is difficult to generate the sufficient amount of the magnetic field on large scales by using the superadiabatic growth within the one-bubble open inflation scenario even if the Higgs mechanism spontaneously breaks gauge and conformal invariances.

Utilizing the obtained mode functions, we have explicitly computed the Wightman function of the $U(1)$ gauge field in terms of the coordinates in the open chart of the de Sitter spacetime, and have compared it with one obtained by other methods. It was found that the leading-order Wightman function in the decoupling ($e \rightarrow 0$) limit is correctly reproduced by the sum of the products of the subcurvature modes without need for introducing supercurvature modes. In consequence we have verified that the supercurvature mode is not needed as a part of a complete set of mode functions of the $U(1)$ gauge field in the decoupling limit.

An interesting observation made in subsection 5.1 is that the existence/absence of supercurvature modes can be strongly related to symmetries of the theory. While a massive scalar field with a sufficiently light mass has a supercurvature mode [26] that survives the massless limit, a theory of a scalar field with shift symmetry does not allow a physical supercurvature mode. This is because the would-be supercurvature mode does not show up in any correlation functions invariant under the shift symmetry. Furthermore, a vector field with a $U(1)$ gauge symmetry does not have a supercurvature mode even when the vector field is given a mass by the Higgs mechanism and thus absorbs a light scalar degree of freedom (the phase of the complex Higgs field). It may be interesting to investigate supercurvature modes of the vector field when we take metric perturbations into account since gravity has the diffeomorphism symmetry, although in the present paper we take account of the effect of gravity only through a curved background. The evaluation of the metric perturbations is beyond the scope of the present paper and we hope to come back to this issue in a future publication.

In this paper, we have assumed several simplifications: (i) the universe during inflationary era after a quantum tunneling is assumed to be well approximated by an exact de Sitter spacetime in the open chart; (ii) the origin of the breaking of the gauge and conformal symmetries and the mass of the vector field is assumed to be the standard Higgs mechanism; (iii) the mass squared $V''_{\Phi}(|\Phi|)$ around the minimum of the Higgs potential is assumed to be large enough so that the mass of the vector field can be considered as constant during inflation. Relaxing some of these assumptions would in principle affect details of our results, although generic features that we have found are expected to remain the same. Furthermore, we have neglected the interactions between the tunneling field and the other fields such as the Higgs field. If we take into account such interactions, then spatially localized, bubble-shaped features may appear [32]. We hope to come back to these issues in the near future.

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A Intrinsic covariant derivative and reduced action

In this appendix, we list some useful formulas which we have used in our calculations. We present the relation between the intrinsic covariant derivatives on the unit 2-sphere and the 4-dimensional covariant derivatives. We first evaluate the Christoffel symbols of the metric $\bar{g}_{\mu\nu}$ defined in eq. (2.9):

$$\Gamma_{ab}^r = \cosh r \sinh r \omega_{ab}, \quad \Gamma_{rb}^a = \tanh r \delta_b^a, \quad (\text{A.1})$$

$$\Gamma_{\phi\phi}^\theta = -\sin\theta \cos\theta, \quad \Gamma_{\theta\phi}^\phi = \cot\theta, \quad \text{otherwise} = 0. \quad (\text{A.2})$$

For the purpose of the $(1+1+2)$ decomposition, we introduce the basis vectors given by

$$u^\mu = \delta_\eta^\mu, \quad n^\mu = \delta_r^\mu, \quad e_\theta^\mu = \frac{1}{\cosh r} \delta_\theta^\mu, \quad e_\phi^\mu = \frac{1}{\cosh r} \delta_\phi^\mu. \quad (\text{A.3})$$

We can evaluate the following equations in terms of these basis vectors:

$$u^\mu \bar{\nabla}_\mu u^\nu = 0, \quad u^\mu \bar{\nabla}_\mu n^\nu = 0, \quad u^\mu \bar{\nabla}_\mu e_a^\nu = 0, \quad (\text{A.4})$$

$$n^\mu \bar{\nabla}_\mu u^\nu = 0, \quad n^\mu \bar{\nabla}_\mu n^\nu = 0, \quad n^\mu \bar{\nabla}_\mu e_a^\nu = 0, \quad (\text{A.5})$$

$$e_a^\mu \bar{\nabla}_\mu u^\nu = 0, \quad e_a^\mu \bar{\nabla}_\mu n^\nu = \tanh r e_a^\nu, \quad (\text{A.6})$$

$$e_\theta^\mu \bar{\nabla}_\mu e_\theta^\nu = e_\theta^\mu \bar{\nabla}_\mu e_\phi^\nu = \frac{1}{\cosh r} \cot\theta e_\phi^\nu, \quad (\text{A.7})$$

$$e_\theta^\mu \bar{\nabla}_\mu e_\theta^\nu = \tanh r n^\nu, \quad e_\phi^\mu \bar{\nabla}_\mu e_\phi^\nu = \tanh r \sin^2\theta n^\nu - \frac{\sin\theta \cos\theta}{\cosh r} e_\theta^\nu, \quad (\text{A.8})$$

where $\bar{\nabla}_\mu$ denotes the covariant derivative with respect to the conformally related 4-dimensional metric $\bar{g}_{\mu\nu}$. With these notations, the 4-dimensional metric can be decomposed as

$$\bar{g}_{\mu\nu} = u_\mu u_\nu - n_\mu n_\nu + \omega_{ab} e_\mu^a e_\nu^b. \quad (\text{A.9})$$

We then derive the explicit relation between the covariant derivative of a two vector on the unit 2-sphere and the 4-dimensional covariant derivative as

$$\frac{1}{\cosh r} X_{a;b} \equiv e_b^\mu \bar{\nabla}_\mu (X_\nu e_a^\nu) - (e_\nu^c e_a^\mu \bar{\nabla}_\mu e_b^\nu) (X_\sigma e_c^\sigma) = e_b^\mu e_a^\nu \bar{\nabla}_\mu (X_c e_\nu^c). \quad (\text{A.10})$$

Using the intrinsic covariant derivative and adopting the convention to denote the projection of tensors as

$$A_\eta \equiv A_\mu u^\mu, \quad A_r \equiv A_\mu n^\mu, \quad \tilde{A}_a \equiv A_\mu e_a^\mu, \quad (\text{A.11})$$

where a runs θ and ϕ , the action (3.2) can be rewritten in terms of the 2-scalars, A_η, A_r , and a 2-vector, \tilde{A}_a , as

$$\begin{aligned} S = & \frac{1}{2} \int dr d\eta d\Omega \left\{ \cosh^2 r (\partial_r A_\eta - \partial_\eta A_r)^2 + \omega^{ab} \left[\partial_r (\cosh r \tilde{A}_a) - A_{r;a} \right] \left[\partial_r (\cosh r \tilde{A}_b) - A_{r;b} \right] \right. \\ & - \omega^{ab} (\cosh r \partial_\eta \tilde{A}_a - A_{\eta;a}) (\cosh r \partial_\eta \tilde{A}_b - A_{\eta;b}) - \frac{1}{2} \omega^{am} \omega^{bn} (\tilde{A}_{a;b} - \tilde{A}_{b;a}) (\tilde{A}_{m;n} - \tilde{A}_{n;m}) \\ & \left. + m_{\tilde{A}}^2 a^2 \cosh^2 r (A_r^2 - A_\eta^2 - \omega^{ab} \tilde{A}_a \tilde{A}_b) \right\}. \quad (\text{A.12}) \end{aligned}$$

Decomposing the 2-vector into the even and odd parity modes (see eq. (3.3)), and expand the quantities in terms of the spherical harmonics $Y_{\ell m}(\Omega)$ (see eqs. (3.4), (3.5)), we then obtain the reduced action (3.13)-(3.8) in the main text.

B Harmonics in open universe

We briefly summarize the formulas for the scalar and vector harmonics on the open universe. To characterize the harmonics in open universe, we introduce the metric on open chart (called region-J ($J = R, L$) hereafter), which is defined by

$$ds_J^2 = a_J^2(\eta_J) \left[-d\eta_J^2 + \gamma_{ij} dx^i dx^j \right] = a_J^2(\eta_J) \left[-d\eta_J^2 + dr_J^2 + \sinh^2 r_J \omega_{ab} d\theta^a d\theta^b \right], \quad (\text{B.1})$$

where $a_J = -1/H \sinh \eta_J$.

B.1 Scalar harmonics

The normalized scalar harmonics, $Y^{p\ell m}$, are the eigen function for the Laplacian operator $\bar{\nabla}^2$ on the 3-dimensional hyperboloid in the region-J:

$$\bar{\nabla}^2 Y^{p\ell m} + (p^2 + 1) Y^{p\ell m} = 0, \quad (\text{B.2})$$

where $\bar{\nabla}_i$ denotes the covariant derivative with respect to the three-dimensional metric γ_{ij} , p is the wave number, ℓ and m denote the angular momentum. The scalar harmonics are expressed in the form

$$Y^{p\ell m}(r_J, \Omega) = f^{p\ell}(r_J) Y_{\ell m}(\Omega), \quad (\text{B.3})$$

where $Y_{\ell m}(\Omega)$ is the spherical harmonic function on the unit two-sphere. The equation for $f^{p\ell}$ is given from eq. (B.2):

$$\left[-\frac{1}{\sinh^2 r_J} \frac{d}{dr_J} \left(\sinh^2 r_J \frac{d}{dr_J} \right) + \frac{\ell(\ell+1)}{\sinh^2 r_J} \right] f^{p\ell}(r_J) = (p^2 + 1) f^{p\ell}(r_J). \quad (\text{B.4})$$

Requiring the regularity at $r_J = 0$, the eigenfunction is given by

$$f^{p\ell}(r_J) \propto \frac{1}{\sqrt{\sinh r_J}} P_{ip-\frac{1}{2}}^{-\ell-\frac{1}{2}}(\cosh r_J) \equiv \mathcal{P}_{p\ell}(r_J), \quad (\text{B.5})$$

where P_ν^μ is the associated Legendre function of the first kind. For the continuous mode ($p^2 > 0$), we fix the normalization factor so that $Y^{p\ell m}$ satisfies

$$\int dr_J d\Omega \sinh^2 r_J Y^{p\ell m}(r_J, \Omega) \overline{Y^{p'\ell'm'}(r_J, \Omega)} = \delta(p - p') \delta_{\ell\ell'} \delta_{mm'}. \quad (\text{B.6})$$

Because the divergent contribution at $p = p'$ comes only from the boundaries of integration at $r_J = \pm\infty$, the integration can be evaluated without investigating the detailed behavior of

eq. (B.5). Using the asymptotic behavior of eq. (B.5) near the boundaries, we have

$$\begin{aligned}
& \int_0^\infty dr_J \sinh^2 r_J \mathcal{P}_{p\ell}(r_J) \overline{\mathcal{P}_{p'\ell}(r_J)} \\
&= \lim_{\epsilon \rightarrow 0} \left[\frac{\Gamma(ip)\Gamma(-ip')}{\Gamma(ip + \ell + 1)\Gamma(-ip' + \ell + 1)} \int_{1/\epsilon}^\infty \frac{dr}{2\pi} e^{i(p-p')r} \right. \\
&\quad \left. + \frac{\Gamma(-ip)\Gamma(ip')}{\Gamma(-ip + \ell + 1)\Gamma(ip' + \ell + 1)} \int_{1/\epsilon}^\infty \frac{dr}{2\pi} e^{-i(p-p')r} \right] \\
&= \frac{\Gamma(ip)\Gamma(-ip)}{\Gamma(ip + \ell + 1)\Gamma(-ip + \ell + 1)} \delta(p - p').
\end{aligned} \tag{B.7}$$

We then have the normalized solution for the continuous mode $f^{p\ell}$ as

$$f^{p\ell}(r_J) = \sqrt{\frac{\Gamma(ip + \ell + 1)\Gamma(-ip + \ell + 1)}{\Gamma(ip)\Gamma(-ip) \sinh r_J}} P_{ip - \frac{1}{2}}^{-\ell - \frac{1}{2}}(\cosh r_J). \tag{B.8}$$

Using the relation between coordinates eq. (2.7) we analytically continue eq. (B.4) to the region-C. We obtain the equation for the analytic-continued $f^{p\ell}$ as

$$\left[-\frac{1}{\cosh^2 r} \frac{d}{dr} \left(\cosh^2 r \frac{d}{dr} \right) - \frac{\ell(\ell + 1)}{\cosh^2 r} \right] f^{p\ell}(r) = (p^2 + 1) f^{p\ell}(r). \tag{B.9}$$

The analytic continuation of the eigenfunction to the region-C are given by

$$f^{p\ell}(r) = \sqrt{\frac{\Gamma(ip + \ell + 1)\Gamma(-ip + \ell + 1)}{i\Gamma(ip)\Gamma(-ip) \cosh r}} P_{ip - \frac{1}{2}}^{-\ell - \frac{1}{2}}(i \sinh r). \tag{B.10}$$

We should note that the Wronskian relation for the Legendre functions leads to the useful formula for the continuous modes [29]:

$$i \cosh^2 r \left\{ \frac{d f^{p\ell}}{dr} \overline{f^{p\ell}} - f^{p\ell} \frac{d \overline{f^{p\ell}}}{dr} \right\} = \frac{2p}{\pi} \sinh(\pi p). \tag{B.11}$$

For the supercurvature mode with the imaginary wave number, the normalization condition in eq. (B.6) is not suitable. We then introduce the supercurvature eigenfunction $f^{\Lambda\ell}$ with $\Lambda = ip$ as the solutions to eq. (B.9). One possible choice of the normalization for the scalar harmonics for the discrete mode, $\mathcal{Y}^{\Lambda\ell m}$, are given by [29]

$$\mathcal{Y}^{\Lambda\ell m}(r, \Omega) = f^{\Lambda\ell}(r) Y_{\ell m}(\Omega), \tag{B.12}$$

$$f^{\Lambda\ell}(r) = \sqrt{\frac{\Gamma(\Lambda + \ell + 1)\Gamma(-\Lambda + \ell + 1)}{2 \cosh r}} P_{\Lambda - \frac{1}{2}}^{-\ell - \frac{1}{2}}(i \sinh r), \tag{B.13}$$

where we fix the normalization factor so that $\mathcal{Y}^{\Lambda\ell m}$ are Klein-Gordon normalized in the region-C, namely

$$\begin{aligned}
& i \cosh^2 r \int d\Omega \left\{ \left(\partial_r \mathcal{Y}^{\Lambda\ell m} \right) \overline{\mathcal{Y}^{\Lambda\ell' m'}} - \mathcal{Y}^{\Lambda\ell m} \left(\partial_r \overline{\mathcal{Y}^{\Lambda\ell' m'}} \right) \right\} \\
&= i \cosh^2 r \left\{ \frac{d f^{\Lambda\ell}}{dr} \overline{f^{\Lambda\ell'}} - f^{\Lambda\ell} \frac{d \overline{f^{\Lambda\ell'}}}{dr} \right\} \int d\Omega Y_{\ell m} \overline{Y_{\ell' m'}} = \delta_{\ell\ell'} \delta_{mm'}.
\end{aligned} \tag{B.14}$$

B.2 Vector harmonics

The normalized (transverse) vector harmonics, $Y_i^{(\lambda)p\ell m}$ with $\lambda = e, o$ being the parity of the harmonics, are eigen functions of the Laplacian operator $\bar{\nabla}^2$ on the 3-dimensional hyperboloid in the region-J:

$$\bar{\nabla}^2 Y_i^{(\lambda)p\ell m} + (p^2 + 2)Y_i^{(\lambda)p\ell m} = 0, \quad \bar{\nabla}^i Y_i^{(\lambda)p\ell m} = 0. \quad (\text{B.15})$$

Apart from the normalization constant, the explicit expression for the vector harmonics can be given by [35]

$$Y_r^{(e)p\ell m} \propto \frac{1}{\sinh r_J} \mathcal{P}_{p\ell}(r_J) Y_{\ell m}(\Omega), \quad (\text{B.16})$$

$$Y_a^{(e)p\ell m} \propto \frac{1}{\ell(\ell+1)} \frac{d}{dr_J} \left[\sinh r_J \mathcal{P}_{p\ell}(r_J) \right] Y_{\ell m;a}(\Omega), \quad (\text{B.17})$$

for the even parity mode of the vector harmonics, and

$$Y_r^{(o)p\ell m} = 0, \quad Y_a^{(o)p\ell m} \propto \sinh r_J \mathcal{P}_{p\ell}(r_J) Y_{\ell m;b}(\Omega) \epsilon^b_a, \quad (\text{B.18})$$

for the odd parity mode of the vector harmonics, where $\mathcal{P}_{p\ell}$ have been defined in eq. (B.5), the colon (:) and ϵ^a_b are the covariant derivative and the Levi-Civita symbol on the unit two-sphere. On a two-dimensional spacetime, the Levi-Civita symbol is a traceless antisymmetric rank-two tensor given by

$$\epsilon_{ab} = \sin \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (\text{B.19})$$

One can verify the following properties:

$$\epsilon_{ca} \epsilon^c_b = -\epsilon_{ac} \epsilon^c_b = \text{diag}(1, \sin^2 \theta), \quad \epsilon_{ab;c} = 0. \quad (\text{B.20})$$

The normalization factor for the continuous mode ($p^2 > 0$) can be fixed so that $Y_i^{(\lambda)p\ell m}$ satisfies

$$\int dr_J d\Omega \sinh^2 r_J \gamma^{ij}(r_J, \Omega) Y_i^{(\lambda)p\ell m}(r_J, \Omega) \overline{Y_j^{(\lambda')p'\ell'm'}(r_J, \Omega)} = \delta(p-p') \delta_{\lambda\lambda'} \delta_{\ell\ell'} \delta_{mm'}. \quad (\text{B.21})$$

As in the case of the scalar harmonics, the divergence contribution to the integral comes only from the boundary of integration at $r_J = \pm\infty$. With a help of the asymptotic behavior of $\mathcal{P}_{p\ell}$, we first evaluate the following integration for the even parity mode of the vector harmonics:

$$\begin{aligned} & \int_0^\infty dr_J \left\{ \mathcal{P}_{p\ell}(r_J) \overline{\mathcal{P}_{p\ell}(r_J)} + \frac{1}{\ell(\ell+1)} \frac{d}{dr_J} \left[\sinh r_J \mathcal{P}_{p\ell}(r_J) \right] \frac{d}{dr_J} \left[\sinh r_J \overline{\mathcal{P}_{p\ell}(r_J)} \right] \right\} \\ &= \lim_{\epsilon \rightarrow 0} \frac{pp'}{\ell(\ell+1)} \left[\frac{\Gamma(ip)\Gamma(-ip')}{\Gamma(1+\ell+ip)\Gamma(1+\ell-ip')} \int_{1/\epsilon}^\infty \frac{dr_J}{2\pi} e^{i(p-p')r_J} \right. \\ & \quad \left. + \frac{\Gamma(-ip)\Gamma(ip')}{\Gamma(1+\ell-ip)\Gamma(1+\ell+ip')} \int_{1/\epsilon}^\infty \frac{dr_J}{2\pi} e^{-i(p-p')r_J} \right] \\ &= \frac{p^2}{\ell(\ell+1)} \frac{\Gamma(ip)\Gamma(-ip)}{\Gamma(ip+1+\ell)\Gamma(-ip+1+\ell)} \delta(p-p'). \end{aligned} \quad (\text{B.22})$$

Hence, the explicit forms of the vector harmonics for the even parity mode are given by

$$Y_r^{(e)p\ell m} = \frac{\sqrt{\ell(\ell+1)}}{p} \frac{1}{\sinh r_J} f^{p\ell}(r_J) Y_{\ell m}(\Omega), \quad (\text{B.23})$$

$$Y_a^{(e)p\ell m} = \frac{1}{\sqrt{\ell(\ell+1)p}} \frac{d}{dr_J} \left[\sinh r_J f^{p\ell}(r_J) \right] Y_{\ell m:a}(\Omega). \quad (\text{B.24})$$

where $f^{p\ell}(r_J)$ have been defined in eq. (B.8). For the odd parity mode of the vector harmonics, we can easily evaluate the normalization condition (B.21) by using eq. (B.7). We then obtain the explicit expression for the odd parity mode of the vector harmonics as

$$Y_r^{(o)p\ell m} = 0, \quad Y_a^{(o)p\ell m} = \frac{1}{\sqrt{\ell(\ell+1)}} \sinh r_J f^{p\ell}(r_J) Y_{\ell m:b}(\Omega) \epsilon_a^b. \quad (\text{B.25})$$

As with the case of the scalar harmonics eq. (B.11), one can verify

$$\begin{aligned} & i \cosh^2 r \int d\Omega \gamma^{ij} \left\{ \left(\partial_r Y_i^{(\lambda)p\ell m} \right) \overline{Y_j^{(\lambda')p\ell' m'}} - Y_i^{(\lambda)p\ell m} \left(\partial_r \overline{Y_j^{(\lambda')p\ell' m'}} \right) \right\} \\ &= i \cosh^2 r \left\{ \frac{d f^{p\ell}}{dr} \overline{f^{p\ell'}} - f^{p\ell} \frac{d \overline{f^{p\ell'}}}{dr} \right\} \delta_{\lambda\lambda'} \int d\Omega Y_{\ell m} \overline{Y_{\ell' m'}} = \frac{2p}{\pi} \sinh(\pi p) \delta_{\lambda\lambda'} \delta_{\ell\ell'} \delta_{mm'}. \end{aligned} \quad (\text{B.26})$$

The vector harmonics for the supercurvature mode with $\Lambda = ip > 0$, $\mathcal{Y}_i^{(\lambda)\Lambda\ell m}$, are described in terms of $f^{\Lambda\ell}$ defined in eq. (B.13) as

$$\mathcal{Y}_r^{(e)\Lambda\ell m} = \frac{\sqrt{\ell(\ell+1)}}{\Lambda} \frac{1}{\cosh r} f^{\Lambda\ell}(r) Y_{\ell m}(\Omega), \quad (\text{B.27})$$

$$\mathcal{Y}_a^{(e)\Lambda\ell m} = -\frac{1}{\sqrt{\ell(\ell+1)\Lambda}} \frac{d}{dr} \left[\cosh r f^{\Lambda\ell}(r) \right] Y_{\ell m:a}(\Omega), \quad (\text{B.28})$$

for the even parity,

$$\mathcal{Y}_r^{(o)\Lambda\ell m} = 0, \quad \mathcal{Y}_a^{(o)\Lambda\ell m} = \frac{1}{\sqrt{\ell(\ell+1)}} i \cosh r f^{\Lambda\ell}(r) Y_{\ell m:b}(\Omega) \epsilon_a^b, \quad (\text{B.29})$$

for the odd parity, where $\mathcal{Y}_i^{(\lambda)\Lambda\ell m}$ are Klein-Gordon normalized in the region-C:

$$\begin{aligned} & i \cosh^2 r \int d\Omega \gamma^{ij} \left[\left(\partial_r \mathcal{Y}_i^{(\lambda)\Lambda\ell m} \right) \overline{\mathcal{Y}_j^{(\lambda')\Lambda\ell' m'}} - \mathcal{Y}_i^{(\lambda)\Lambda\ell m} \left(\partial_r \overline{\mathcal{Y}_j^{(\lambda')\Lambda\ell' m'}} \right) \right] \\ &= i \cosh^2 r \left\{ \frac{d f^{\Lambda\ell}}{dr} \overline{f^{\Lambda\ell'}} - f^{\Lambda\ell} \frac{d \overline{f^{\Lambda\ell'}}}{dr} \right\} \int d\Omega Y_{\ell m} \overline{Y_{\ell' m'}} = \delta_{\lambda\lambda'} \delta_{\ell\ell'} \delta_{mm'}, \end{aligned} \quad (\text{B.30})$$

where we have used eq. (B.14).

C Klein-Gordon norm

In this section, we give the explicit expression for the Klein-Gordon norm incorporating the non-dynamical field, following and extending [36]. Let us begin with the system of

the m -physical degrees of freedom ϕ^A ($A = 1, \dots, m$) and n -auxiliary variables φ^α ($\alpha = m+1, \dots, m+n$). The discretized Lagrangian we will consider here is given by

$$L = \frac{1}{2} G_{AB} \left(\dot{\phi}^A - f^A_C \phi^C - \tilde{f}^A_\alpha \varphi^\alpha \right) \left(\dot{\phi}^B - f^B_D \phi^D - \tilde{f}^B_\beta \varphi^\beta \right) - \frac{1}{2} V_{AB} \phi^A \phi^B - M_{A\alpha} \phi^A \varphi^\alpha - \frac{1}{2} \tilde{V}_{\alpha\beta} \varphi^\alpha \varphi^\beta. \quad (\text{C.1})$$

We recast the Lagrangian in terms of the matrix description as

$$L = \frac{1}{2} \left(\dot{\phi}^T - \phi^T \mathbf{f}^T - \varphi^T \tilde{\mathbf{f}}^T \right) \mathbf{G} \left(\dot{\phi} - \mathbf{f} \phi - \tilde{\mathbf{f}} \varphi \right) - \frac{1}{2} \phi^T \mathbf{V} \phi - \phi^T \mathbf{M} \varphi - \frac{1}{2} \varphi^T \tilde{\mathbf{V}} \varphi. \quad (\text{C.2})$$

The equation for φ can be obtained by varying the Lagrangian (C.2) as

$$\varphi = \left(\tilde{\mathbf{f}}^T \mathbf{G} \tilde{\mathbf{f}} - \tilde{\mathbf{V}} \right)^{-1} \left[\tilde{\mathbf{f}}^T \mathbf{G} \dot{\phi} + \left(\mathbf{M}^T - \tilde{\mathbf{f}}^T \mathbf{G} \mathbf{f} \right) \phi \right]. \quad (\text{C.3})$$

Substituting the constraint equation (C.3) into the Lagrangian (C.2), after lengthy calculation, we obtain the reduced Lagrangian as

$$L = \frac{1}{2} \left(\dot{\phi}^T - \phi^T \mathbf{f}_{\text{eff}}^T \right) \mathbf{G}_{\text{eff}} \left(\dot{\phi} - \mathbf{f}_{\text{eff}} \phi \right) - \frac{1}{2} \phi^T \mathbf{V}_{\text{eff}} \phi, \quad (\text{C.4})$$

where

$$\mathbf{f}_{\text{eff}} = \mathbf{f} - \tilde{\mathbf{f}} \tilde{\mathbf{V}}^{-1} \mathbf{M}^T, \quad (\text{C.5})$$

$$\mathbf{G}_{\text{eff}} = \mathbf{G} - \mathbf{G} \tilde{\mathbf{f}} \left(\tilde{\mathbf{f}}^T \mathbf{G} \tilde{\mathbf{f}} - \tilde{\mathbf{V}} \right)^{-1} \tilde{\mathbf{f}}^T \mathbf{G}, \quad (\text{C.6})$$

$$\mathbf{V}_{\text{eff}} = \mathbf{V} - \mathbf{f}^T \mathbf{G} \mathbf{f} + \left(\mathbf{M} - \mathbf{f}^T \mathbf{G} \tilde{\mathbf{f}} \right) \left(\tilde{\mathbf{f}}^T \mathbf{G} \tilde{\mathbf{f}} - \tilde{\mathbf{V}} \right)^{-1} \left(\mathbf{M}^T - \tilde{\mathbf{f}}^T \mathbf{G} \mathbf{f} \right) + \mathbf{f}_{\text{eff}}^T \mathbf{G}_{\text{eff}} \mathbf{f}_{\text{eff}}. \quad (\text{C.7})$$

We have used the useful formula as

$$\mathbf{G}_{\text{eff}} \mathbf{f}_{\text{eff}} = \mathbf{G} \mathbf{f} + \mathbf{G} \tilde{\mathbf{f}} \left(\tilde{\mathbf{f}}^T \mathbf{G} \tilde{\mathbf{f}} - \tilde{\mathbf{V}} \right)^{-1} \left(\mathbf{M}^T - \tilde{\mathbf{f}}^T \mathbf{G} \mathbf{f} \right). \quad (\text{C.8})$$

We now promote ϕ to operators $\hat{\phi}$, and expand $\hat{\phi}$ by mode functions $\{\phi_{\mathcal{N}}, \overline{\phi_{\mathcal{N}}}\}$, which is expressed as

$$\hat{\phi} = \sum_{\mathcal{N}} \left(a_{\mathcal{N}} \phi_{\mathcal{N}} + a_{\mathcal{N}}^\dagger \overline{\phi_{\mathcal{N}}} \right), \quad (\text{C.9})$$

where $\hat{a}_{\mathcal{N}}$ and $\hat{a}_{\mathcal{N}}^\dagger$ are the annihilation and creation operators, respectively, we assume that $\{\phi_{\mathcal{N}}, \overline{\phi_{\mathcal{N}}}\}$ forms a complete set of linear independent solutions of the equation of motion. The quantum fluctuations of the field are described by the vacuum state, which is annihilated by the annihilation operator, $\hat{a}_{\mathcal{N}}|0\rangle = 0$. According to [36], the discretized KG norm for the reduced Lagrangian (C.4) can be defined by

$$(\phi_{\mathcal{N}}, \phi_{\mathcal{M}})_{\text{KG}} = -i \left\{ \phi_{\mathcal{N}}^T \mathbf{G}_{\text{eff}} \left(\overline{\dot{\phi}_{\mathcal{M}}} - \mathbf{f}_{\text{eff}} \overline{\phi_{\mathcal{M}}} \right) - \left(\dot{\phi}_{\mathcal{N}}^T - \phi_{\mathcal{N}}^T \mathbf{f}_{\text{eff}}^T \right) \mathbf{G}_{\text{eff}} \overline{\phi_{\mathcal{M}}} \right\}. \quad (\text{C.10})$$

With the help of eqs. (C.3) and (C.8), the KG norm for the physical degrees of freedom ϕ can be reduced to the simple form in terms of the auxiliary variables φ as

$$(\phi_N, \phi_M)_{\text{KG}} = -i \left\{ \phi_N^T G \left(\overline{\dot{\phi}_M} - \overline{f \phi_M} - \overline{\tilde{f} \varphi_M} \right) - \left(\dot{\phi}_N^T - \phi_N^T f^T - \varphi_N^T \tilde{f}^T \right) G \overline{\phi_M} \right\}, \quad (\text{C.11})$$

that is,

$$(\phi_N, \phi_M)_{\text{KG}} = -i G_{AB} \left\{ \phi_N^A \left(\overline{\dot{\phi}_M^B} - f^B_D \overline{\phi_M^D} - \tilde{f}^B_\beta \overline{\varphi_M^\beta} \right) - \left(\dot{\phi}_N^A - f^A_C \phi_N^C - \tilde{f}^A_\alpha \varphi_N^\alpha \right) \overline{\phi_M^B} \right\}, \quad (\text{C.12})$$

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